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THE COVARIANCE ANALYSIS OF MULTIPLE CLASSIFICATION TABLES WITH UNEQUAL SUBCLASS NUMBERS

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Data which can be classified simultaneously in two or more ways with unequal numbers in the various subclasses are commonly found in many kinds of experimental work, particularly where unequal numbers are characteristic of the parent populations. Frequently, too, the attribute under study may be associated with one or more independent variables which exhibit

Table 1. *Numbers, weights, ages and inbreeding of lambs classed according to age of dam and type of birth*

Type of birth	*Attribute*			*Age of dam* Mature dams D_1	Young dams D_2	*Total*
Single lambs (T_1)	Number			179	105	284
	Weight	(pounds)	Y	15,187	7,740	22,927
	Age	(days)	A	21,392	12,639	34,031
	Inbreeding	(percent)	P	1,768.9	1,376.9	3,145.8
Twin lambs (T_2)	Number			117	18	135
	Weight	(pounds)	Y	8,422	1,138	9,560
	Age	(days)	A	14,141	2,150	16,291
	Inbreeding	(percent)	P	1,098.8	72.4	1,171.2
Twin lambs reared singly (T_3)	Number			56	3	59
	Weight	(pounds)	Y	4,346	213	4,559
	Age	(days)	A	6,716	366	7,082
	Inbreeding	(percent)	P	634.6	66.2	700.8
Total	Number			352	126	478
	Weight	(pounds)	Y	27,955	9,091	37,046
	Age	(days)	A	42,249	15,155	57,404
	Inbreeding	(percent)	P	3,502.3	1,515.5	5,017.8

$S(Y)^2 = 2{,}963{,}362$ $\quad$ $S(YA) = 4{,}452{,}731$

$S(A)^2 = 6{,}904{,}328$ $\quad$ $S(YP) = 378{,}947.0$

$S(P)^2 = 92{,}156.64$ $\quad$ $S(AP) = 602{,}971.6$

continuous variation. When independent variables and multiple classifications with unequal subclass numbers occur in the same group of data, some problems in analysis arise which have not been extensively dealt with. This report is intended to give an outline of a method suitable for the analysis of data with the above properties.

The lamb weights summarized in Table 1 may be used to illustrate some of the problems and the method of analysis. The lambs are grouped according to age of dam into two classes and according to type of birth into three classes, making a total of six subclasses. The extreme disproportion in subclass numbers is due to the fact that mature ewes normally produce a greater proportion of twin lambs than do young ewes. In addition to the two major classifications, two independent variables, the age in days at weighing and percent inbreeding may be associated with weight of the lambs.

The practical statistician is usually concerned with obtaining efficient and unbiased estimates of certain population parameters from a given body of data, the appropriate method of analysis depending upon the parameters to be estimated. In the present case, the method of analysis is specifically designed to eliminate from each estimate possible inequalities due to the other factors insofar as this is possible. For example, the difference in average weight between lambs having mature and young dams is 7.27 pounds. Evidently this difference does not provide an unbiased estimate of the effect of age of dam unless we are prepared to ignore possible discrepancies due to the disproportionate subclass numbers and to differences in age and inbreeding.

Yates (2) presented a method of applying least squares procedure to the analysis of multiple classification tables with unequal numbers in the subclasses. This method, frequently described as the method of fitting constants, can be extended without difficulty to include independent variables. In the present example, one constant must be fitted for the Y-intercept, two for age of dam, three for type of birth, and one each for age and percent inbreeding. Each observation is defined by the equation:

(1) $Y = a + d_1D_1 + d_2D_2 + t_1T_1 + t_2T_2 + t_3T_3 + b_aA + b_pP + E.$

Here the lower case letters represent the constants to be fitted, and the capital letters are defined as follows: D_1 and D_2 take the values 1 and 0, or 0 and 1, depending upon whether an observation lies in the mature dam group or in the young dam group; T_1, T_2, and T_3 take the values 1, 0, and 0, or 0, 1, and 0, or 0, 0, and 1, depending upon where an observation lies with respect to type of birth; A and P are the values for age and percent inbreeding which are associated with that particular observation, while E represents the error. The least squares procedure provides estimates of the constants such that $S(E)^2$ is a minimum as compared with any other set of additive and linear constants.

Summing equation 1 over the 478 observations in Table 1 we obtain the equation:

(2) $478a + 352d_1 + 126d_2 + 284t_1 + 135t_2 + 59t_3 + 57{,}404b_a + 5017.8b_p = 37{,}046,$

which can be used subsequently in estimating the Y-intercept *a*. The least squares method of setting up the simultaneous equations is to multiply equation 1 by each variable (D_1, D_2, T_1, etc.) summing over all observations. Using the data in Table 1, this procedure gives the following equations:

(3) $352a + 352d_1 + 0d_2 + 179t_1 + 117t_2 + 56t_3 + 42249b_a + 3502.3b_p = 27{,}955$

(4) $126a + 0d_1 + 126d_2 + 105t_1 + 18t_2 + 3t_3 + 15{,}155b_a + 1515.5b_p = 9{,}091$

(5) $284a + 179d_1 + 105d_2 + 284t_1 + 0t_2 + 0t_3 + 34031b_a + 3145.8b_p = 22{,}927$

(6) $135a + 117d_1 + 18d_2 + 0t_1 + 135t_2 + 0t_3 + 16291b_a + 1171.2b_p = 9{,}560$

(7) $59a + 56d_1 + 3d_2 + 0t_1 + 0t_2 + 59t_3 + 7082b_a + 700.8b_p = 4{,}559$

(8) $57404a + 42249d_1 + 15155d_2 + 34031t_1 + 16291t_2 + 7082t_3 + 6{,}904{,}328b_a + 602{,}971.6b_p = 4{,}452{,}731$

(9) $5017.8a + 3502.3d_1 + 1515.5d_2 + 3145.8t_1 + 1171.2t_2 + 700.8t_3 + 602{,}971.6b_a + 92156.64b_p = 378{,}947.0$

The equations having the leading terms d_1 and d_2 can be eliminated simultaneously from

the other equations, as can those having the leading terms t_1, t_2, and t_3. By successive elimination the equations can be reduced to the following:

$$\begin{array}{lllll}
(3a) & 83.821d_1 - & 83.821d_2 = & 859.379 & \\
(4a) & -83.821d_1 + & 83.821d_2 = & -859.379 & \\
(5a) & 104.550t_1 - & 73.281t_2 - & 31.269t_3 = & 1189.722 \\
(6a) & -73.281t_1 + & 91.737t_2 - & 18.456t_3 = & -1108.822 \\
(7a) & -31.269t_1 - & 18.456t_2 + & 49.725t_3 = & -80.900 \\
(8a) & 10{,}472.218b_a = & 5130.711 & & \\
(9a) & 38{,}498.842b_p = & -10{,}639.150 & &
\end{array}$$

It is evident that equations 3a and 4a are not independent since their sum is zero. This is consistent with the fact that only 1 degree of freedom exists for age of dam. In addition, equations 5a, 6a, and 7a add to zero, this being consistent with the 2 degrees of freedom for type of birth. Hence an additional relation must be established between d_1 and d_2 and between t_1, t_2 and t_3 to proceed with the analysis. We may take $d_1 + d_2 = 0$ and $t_1 + t_2 + t_3 = 0$, using the appropriate relation to replace one of the above equations in solving for the respective constants. With these restrictions, estimates of the constants are as follows:

$$\begin{array}{ll}
d_1 = 5.12628 & t_3 = 0.06773 \\
d_2 = -5.12628 & b_a = 0.48994 \\
t_1 = 6.67416 & b_p = -0.27635 \\
\multicolumn{2}{c}{t_2 = -6.74192}
\end{array}$$

These estimates being obtained, equation 2 may be solved, the estimate of a being 17.07158.

Unbiased and efficient estimates of the differences between the main classes, with other factors constant, can now be calculated. For example, the difference in weight between lambs having mature and young dams is $d_1 - d_2 = 10.25$ pounds. The difference between single and twin lambs is $t_1 - t_2 = 13.42$ pounds, and that between single lambs and twins reared singly is $t_1 - t_3 = 6.61$ pounds. The constants b_a and b_p are partial regression coefficients measuring the average change in weight associated with one unit increase in age and percent inbreeding, respectively.

The problem of setting up tests of significance logically follows that of estimation. The sum of the squared deviations from the general mean is

$$2{,}963{,}362 - (37{,}046)^2/478 = 92{,}219.5$$

The total reduction due to the several sources of variation including both direct and combined effects is $b_1S(X_1Y) + b_2S(X_2Y) + \ldots - (SY)^2/n$ with the usual least squares procedure. Applying this to the present example wherein $S(X_1Y)$ is given by the right hand terms of equations 2 to 9, we have $17.07158(37{,}046) + 5.12628(27{,}955) + \ldots - (37{,}046)^2/478 = 23{,}716.9$. The direct reduction associated with the ith variable is $b_iS(x_iy)$, where $S(x_iy)$ is the right hand term of the ith normalized equation obtained by eliminating the association with other variables. Applying this principle to the normalized equations for the D, T, A and P variables (equations 3a to 9a), the sum of squares directly associated with each of the major sources of variation is as follows:

$$\begin{array}{lr}
\text{Age of dam} = (5.12628)\ (859.379) + (-5.12628)\ (-859.379) & = 8{,}810.8 \\
\text{Type of birth} = (6.67416)\ (1189.722) + (-6.74192)\ (-1108.822) & \\
\qquad + (0.06776)\ (-80.900) & = 15{,}410.5 \\
\text{Age} = (0.48994)\ (5130.711) & = 2{,}513.7 \\
\text{Percent inbreeding} = (-0.27635)\ (-10{,}639.150) & = 2.940.1 \\
\hline
\quad \text{Reduction in sum of squares due to direct effects} & = 29{,}675.1
\end{array}$$

The direct reduction due to a given source is the difference between the total reduction

with all variables included and that which would be obtained if that particular variable were ignored altogether. This may be illustrated by considering one of several alternative methods of computing the total reduction:

Age of dam (other sources ignored)	4,899.7
Type of birth (age of dam fixed)	13,496.9
Age (age of dam and type of birth fixed)	2,382.9
Inbreeding (age of dam, type of birth and age fixed)	2,940.1
Total reduction in sum of squares	23,719.6

The difference between the present figure and that given above is due to small errors in rounding the regression coefficients.

In the usual least squares procedure dealing with continuous variables, the total reduction will always be equal to or greater than the sum of the direct effects, but this may or may not be true of discrete classifications with unequal subclass numbers. In this particular example, a part of the true difference between ages of dam or types of birth is concealed in the marginal totals because the twin lambs are proportionately more numerous in the mature dam group. This is illustrated by the reduction in the sum of squares due to age of dam; the reduction with other sources ignored is 4,899.7, while with other sources fixed it is 8,810.8. Differences in average age and percent inbreeding evidently contribute somewhat to the discrepancy but their effects are much less important than the disproportionate subclass numbers.

If we are content to assume without further question that equation 1 is adequate to describe the manner in which the main effects combine, the analysis of variance can be set up as shown in Table 2. The difference between the total sum of squares and the total reduction provides an error term for testing the significance of the direct reduction in variance due to each of the main sources. It is evident that all of the main effects are highly significant sources of variation, age of dam and type of birth being more important than age and percent inbreeding.

Table 2. *Analysis of variance for weight of lambs*

Source of variation	*Degrees of freedom*	*Sum of squares*	*Mean squares*
Total	477	92,219.5	....
Total reduction	5	23,716.9	....
Error	472	68,502.6	145.1
Direct effects			
Age of dam	1	8,810.8	8,810.8
Type of birth	2	15,410.5	7,705.2
Age	1	2,513.7	2,513.7
Inbreeding	1	2,940.1	2,940.1

In some cases we may wish to examine the data more closely with respect to their conformance to the rules prescribed in equation 1. This equation is about as simple as any which can be expected to fit the data (since all of the main effects have been found significant) but no assurance is provided that the main effects, and the relations between them, are actually as simple as described therein. For example, the assumption is made in setting up equation 1 that differences in weight due to age of dam and type of birth combine additively, or without interaction. Yates (2) pointed out that the method of fitting constants provided a valid test of the interaction, but not of the main effects where interaction exists. He suggested the method of weighted squares of means for the analysis of data with real interaction. In the present case, we may compare the actual subclass means with "expected" means derived from the constants previously calculated, testing in this manner the interaction between age of dam and type

of birth which is assumed nonexistent in equation 1. The actual and expected subclass means and the difference between them are given in table 3. The sum of squares due to interaction may be obtained by squaring each difference, multiplying by the appropriate subclass number, and summing over the 6 subclasses. Since the sum of squares for interaction is 108.1 and this source of variation has 2 degrees of freedom, the mean square is less than the experimental error in table 2. Hence, the evidence from the data confirms, or at least does not refute, the hypothesis of an additive relation between the main effects for age of dam and type of birth.

Table 3. *Actual and expected mean weights in the subclasses*

Type of birth	*Mean*	*Mature dams*	*Young dams*
Single lambs	Number	179	105
	Actual	84.844	73.714
	Expected	84.692	73.970
	Difference	0.152	—0.256
Twin lambs	Number	117	18
	Actual	71.983	63.222
	Expected	72.076	62.606
	Difference	—0.093	0.616
Twin lambs reared singly	Number	56	3
	Actual	77.607	71.000
	Expected	77.891	65.687
	Difference	—0.284	5.313

The constants fitted for age and for inbreeding are averages of the individual linear regressions within the six subclasses. We may wish to examine more fully whether the individual regressions for age or for percent inbreeding differ significantly among themselves, or whether a curvilinear regression would provide a significantly better fit than a linear regression. Since the appropriate procedures for these tests are similar to those used for the same purposes in the usual covariance analysis they need not be considered in detail.

Snedecor and Cox (1) summarized and compared the several methods suitable for the analysis of multiple classication tables with unequal subclass numbers. They pointed out that Yates' method of fitting constants and conventional methods of analysis of variance yield the same results when the subclass numbers are equal. The method of fitting constants, extended to include independent variables with continuous distributions, also gives the same results with equal subclass numbers as do the conventional methods of covariance analysis. The method can be extended to include as many classifications or as many independent variables as desired, simply by setting up additional simultaneous equations for the additional classifications or variables.

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STATISTICAL METHODS IN CEREAL CHEMISTRY

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Cereal chemistry is essentially a biological science and as such the possible uses of statistical methods are numerous. Since this science deals very frequently with differentiating varieties of field crops for quality characteristics, many of the techniques developed for field plot work are applicable, and there has been a considerable development along this line within recent years.

The analysis of variance is probably the most frequently used statistical tool in cereal chemistry research (13). Applications vary from simple examples such as the comparison of the mean squares for weight per bushel, between and within grades (12), to complex examples involving several factors. A typical example of the latter is given in the report of a study of variability in experimental baking (11). The three factors affecting variability were flours, laboratories, and baking formulas. An outline of one of the analyses is as follows:

	Degrees of freedom
Between flours	4
Between laboratories	2
Between formulas	1
Interactions:	
Flours × laboratories	8
Flours × baking formulas	4
Laboratories × baking formulas	2
Error and triple interaction	727

The triple interaction was found to be insignificant and was combined with the error. This procedure is not to be followed as a general practice but it did not influence the present results. This example illustrates the principle of factorial experimentation as applied to cereal chemistry.

Another experimenter might consider it desirable to resort to methods more appropriate to an elementary demonstration in a physics laboratory in which one factor is varied at a time, with the others held constant. Thus, as the first phase of the investigation several laboratories might have tested the same flour by the same baking formula, and if the laboratories obtained essentially similar results it might be concluded that future figures on loaf volume from the various laboratories would be comparable. Actually, such a conclusion would be correct only under exactly similar conditions of experimentation, that is, with the same flour and the same baking formula. Geddes *et al* (11) have demonstrated that an interaction exists between laboratories and baking formulas, so that the above hypothetical experiment would have missed this extremely important result. The demonstration of an interaction in this instance means that if the different laboratories are using different baking formulas, their results are not entirely comparable. Experiments in which all except one of the factors are held constant are so limited in scope that the results are frequently of no practical value.

A further advantage of factorial experiments is that the interactions can be submitted to a valid test of significance. One measure of interaction between laboratories and baking formulas could be obtained from two separate experiments, one with one baking formula and the other with a second baking formula. The variation in the mean difference between the results of the two formulas at the various baking laboratories would be a measure of interaction, but such a measure of interaction could not always be submitted to a test of significance. Assuming three laboratories, each baking ten loaves, we would have the following analysis of variance for each experiment:

		D.F.
Formula No. 1	Between laboratories	2
	Within laboratories	27
Formula No. 2	Between laboratories	2
	Within laboratories	27

A combined analysis would give

	D.F.
Between formulas	1
Between laboratories	2
Interaction	2
Error	54

in which the error term combines the sum of squares and degrees of freedom within laboratories from the previous separate analyses. If the experimental conditions were exactly comparable this procedure would be justified but frequently conditions are not comparable and the errors differ for the two experiments. In such cases the customary test of significance from a direct application of the analysis of variance is not valid. Cochran (3) discusses this situation in connection with agricultural experiments.

A study on the protein content of bulk wheat (16) concerned the relative importance of various sources of error leading to disagreement between determinations on the same wheat in different laboratories. The factors studied were heterogeneity of bulk wheat, sampling error, variation in cleaning procedure, differences in grinding methods, and analytical error. The variance for grinding, for example, was evaluated and from it were removed the variations due to analytical error and to heterogeneity of wheat. The resulting standard error represented the variation due purely to grinding. The other factors were handled similarly to obtain a basis for ready interpretation. The authors conclude in one instance "that little is gained in precision by carrying out replicated analyses of the same sub-sample of ground wheat. Improvement in results should rather be sought in better sampling and cleaning techniques and in a more uniform grinding procedure."

A similar investigation on alimentary pastes (5) was designed to increase the reproducibility of micro-tests of pastes made from durum wheat flour. The factors were absorption, mixing temperature, mixing time, sheeting temperature, sheeting (number of times), pressure of sheeting rolls and time in the press. Each processing factor was studied at different levels, so that their interactions could be examined. Most of the first order interactions exceeded either the 5 percent or 1 percent levels of significance, showing that the effect of a change in one processing factor depended not only on the level of the factor in question but also on the level, at the time, of the other factors in the experiment. An exceedingly interesting technique employed here was the use of graphs to demonstrate the meaning of the significant first order interactions.

Correlation and regression techniques are applied quite commonly to problems in cereal chemistry. Simple applications have been used frequently but in studies of more than two variates, the investigators have resorted to the method of partials and multiples. A typical study (12) involved the correlation bteween the total flour yield of wheat, with weight per bushel, and four forms of physical damage to the kernels. From the simple correlation coefficients, all the partials were calculated using total flour yield as the dependent variable. Finally the multiple correlation was determined between weight per bushel and the four forms of damage. All simple correlations of total flour yield with each of the other characters were significant beyond the 5 percent level. One of the partial correlations, however, was less than the 5 percent level. This coefficient, which measured the association of green kernels with flour yield when the other variables were held constant, suggests "that green kernels have somewhat less influence on flour yields than the other classes of damaged kernels." Had the data in this experiment been representative of more than one crop year, the influence of the various forms of damaged kernels might have been measured from the partial regression coefficients. In this way estimates would have been obtained of the change in total yield of flour per unit change in any one of the independent variables while the others were held constant. The significance of each coefficient could have been determined with a "t" test, and that of their multiple effect by an "F" test (20).

The technique of covariance is proving extremely useful. Generally the object of the method is to divide heterogeneous correlation effects into homogeneous groups. Due to the type of material with which cereal chemists work, most correlation studies could use covariance to determine whether the results were obtained from homogeneous populations. In one study (19, 14) total nitrogen was correlated with the saccharifying activity of the malt extracts for twelve varieties of barley grown at twelve stations distributed across Canada. In the preliminary analysis the correlations for stations, varieties, and error were

0.959, —0.024, and 0.450, respectively. The station correlation was determined from the 12 paired means for the two characters, where each was the mean of all varieties at one station. The variety correlation was determined from the 12 paired means for each variety over all stations. The correlation for the error or residual was that arising within stations after the variety effect had been removed.

The most striking feature of this analysis is the wide discrepancy between the correlations of the station means and of the variety means. There was an evident relation between the total nitrogen and saccharifying activity as the total nitrogen changed from station to station. Although the station correlation represents only 10 degrees of freedom, it is highly significant. The variety correlation is quite insignificant. The genetic factors causing differences between the varieties in the total nitrogen content of the grain did not have a corresponding effect on the saccharifying activity of the malt extract. This example of heterogeneity in a total covariance illustrates the necessity for separating it into its component parts.

The analysis of covariance is also of importance in testing the homogeneity of regressions. In the above study, for example, the variety coefficients were tested for heterogeneity. A single regression was computed from the total sums of squares and products within all varieties, the residual sum of squares representing the variation for 131 degrees of freedom. When an individual regression was fitted for each variety, the residual sum of squares about the 12 lines represented the variation for only 120 degrees of freedom. The eleven degrees of freedom in the difference measured the variation of the twelve individual regression coefficients about the "average" regression for all varieties. The ratio of these last two variances answered the question as to whether or not all varieties showed the same relation between nitrogen content and saccharifying activity. By this method it is possible to compare correlations or regressions that are in themselves correlated.

Covariance may be used in controlling concomitant errors (7) in studies on cereal chemistry. For example, in comparing varieties for protein content when samples are taken from a wide area, factors such as rainfall or nitrogen content of the soil contribute to the variation in protein. The effect of one or both factors could be removed by covariance with a consequent improvement in the precision of the test. The method has been applied by Crampton and Hopkins (4) and by Eden and Fisher (6).

When an experiment involves several levels of a factor, such as different quantities of bromate in a baking formula, the sum of squares due to levels may be separated into linear, quadratic, cubic, etc. effects (10, 14, 17). In an experiment on loaf volume using four levels of bromate and five varieties, an analysis could be made as follows:

		D.F.
Varieties		4
Treatments —	linear effects	1
	quadratic effects	1
	cubic effects	1
Interactions —	linear effects × varieties	4
	quadratic effects × varieties	4
	cubic effects × varieties	4

If the experimental results were to fall mainly on a straight line, the greater portion of the sum of squares for treaments would be contained in the sum of squares for the linear effect. The sum of squares for quadratic effects representing the additional degree of freedom in fitting a curve of the second degree would be significantly larger than the error if there were a definite trend or curvature in one direction. A similar approach would apply to the cubic and other effects of higher order. Possibly the most interesting application of this method is in testing the significance of the interaction components. The variance of linear effects × varieties, for example, measures the consistency of the slope among varieties.

The method of fitting polynomials when the independent variable occurs at equal intervals and with equal weights (7, 10), is proving of considerable value. In one study (1), the loaf volume of bread from wheat flours was determined for equal intervals of mixing time of the dough. In early experiments, two loaves were baked for each of three mixing times, six or seven loaves being the maximum num-

ber that could be handled for each kind of flour. This did not give a sufficient number of points for fitting curves which would distinguish clearly between the flour types in respect to mixing tolerance. The technique was changed to using seven mixing times, baking one loaf for each, and fitting a polynomial to the seven points. The results indicated a definite quadratic trend for most flours and the quality characteristics of the flour types were much more obvious. The rapidity of this method is a further advantage. It is therefore of considerable value in designing experiments that are expected to show trends, to obtain the data in such a way that the method can be applied.

The determination of a regression integral involving a regression function varying continuously with time (9) has been employed to advantage in certain cereal chemistry studies. In a study of the effect of the amount and distribution of rainfall on the protein content of wheat (18), the results were obtained by this technique. The data on rainfall and protein covered seven stations and a period of fourteen years. The rainfall data for each year were divided into 25 five-day periods.

The first step in the method was to express the amount and distribution of rainfall at each station for each year by the six numerical coefficients of a fifth degree polynomial function of the time:

$$p_0T_0 + p_1T_1 + p_2T_2 + p_3T_3 + p_4T_4 + p_5T_5$$

the six terms being mutually orthogonal. Then by virtue of the orthogonal properties of the functions T_r, the regression integral of protein upon rainfall could itself be expressed as a fifth degree polynomial function of time involving the partial regression coefficients of protein upon the distribution coefficients P_r. The final equation showed the effect on protein of an additional unit of rainfall at any period in the full interval.

The statistical methods now available to the cereal chemist provide him with a valuable tool with which he may reap greater returns for his laboratory efforts. In any group of observations there is only so much information and no more. If efficient statistical methods are applied, all of the relevant information will be extracted from the data, whereas improper statistical analysis would lead to loss of some of the information and possibly to inaccurate conclusions. Furthermore, one group of data may contain more useful information than another group which required equal time and effort. For this reason it is important that an investigation be approached statistically at the very outset—when the experiment is being designed. While the more complex aspects of a statistical analysis may be left to the professional statistician, the cereal chemist must understand the logic of the methods being applied, for his understanding of the principles of design and randomization and the validity of tests is of prime importance when interpreting the final results (8, 2, 21).

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INCOMPLETE-BLOCK DESIGN ADAPTED TO PAIRED TESTS OF MOSQUITO REPELLENTS

F. M. WADLEY

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The knowledge of mosquito repellents has advanced rapidly in the last few years, and testing methods have received considerable attention. The writer has studied variability in determinations of relative effectiveness of repellents made at several places, especially the determinations made by the Bureau of Entomology and Plant Quarantine at Orlando, Fla. The testing methods, involving the exposure of treated arms of human subjects to adult mosquitos, result in a series of records of individual protection periods. The exposures are not continuous but are made at short intervals. This is not a serious statistical handicap, however, with those compounds giving long-time protection.

Early in the studies it was apparent that dates of testing and, especially, subjects usually showed real variability. There may well be variation due to interaction of date and subject also; no tests were made which gave information on this. The variance between right and left arms of a subject at a given time seemed to be as low as could be expected with the methods used, but even this variance was considerable. Date and subject variation must be expected in practical use, but for close comparisons it is well to limit the variance to the paired-arm level. Accordingly many tests were made of the more promising substances paired with the standard repellent. This method had its usual drawbacks—that a large proportion of the work consisted in repeating tests of the standard, and that two new substances had to be compared through the standard, with a loss of precision.

Where several substances of already proved value were to be compared, it was suggested that the "balanced incomplete block" plan be tried (Yates 1940). In this case the "block" had to consist of the 2 arms of a subject on a given date, and hence could contain only 2 units or "plots." A trial was made by F. A. Morton of the Orlando laboratory in 1945, using 5 such substances and the standard. Five subjects were used to form 5 complete replications. Each subject was used on 3 dates, which made a total of 15 blocks. The 6 materials were tested once in every pair combination. Thus in Yates' notation, $k = 2$, $v = 6$, $r = 5$, $b = 15$, $\lambda = 1$. The mosquito used was the standard test species, *Aedes aegypti*, and time to the second bite was the criterion. The analysis was carried out by the method of Yates (1940) for the case where groups of blocks form complete replications.

Six such experiments were carried on. In all of them real differences between materials were found. In each experiment there was a real difference between replications (subjects).

In some of the experiments there was no significant variation between blocks after adjusting for materials and replications; in others block variation occurred. Where there is a real variation between dates there will be a significant variance between blocks within replications, after allowance is made for variance between replications and materials. Hence absence of such block variance indicates absence of differences between dates.

An analysis of an experiment with real block variation is summarized as follows:

	Degrees of freedom	*Sum of squares*	*Mean square*
Materials (ignoring blocks)	5	67,631	
Replications (subjects)	4	57,215	
Blocks (eliminating materials and replications)	10	100,251	10,025
Error (intra-block)	10	18,260	1,826

Adjusted material means were 242 (*the standard*), 342, 355, 156, 267, and 238 minutes, compared with unadjusted values of 247, 329, 320, 189, 267, and 248, respectively. The least significant difference between adjusted means was 74 minutes, at the 5 percent level. The variance of a material total is here

$[kr(v-1)]/[Wv(k-1)+W'(v-k)]$,

where W is the reciprocal of the error variance and W′ is $(r-1)/[(rx \text{ block variance}) - (\text{error variance})]$. From this variance the standard error of a mean difference can easily be determined.

The gain of efficiency in this plan over that of randomized complete blocks is about 90 per cent; recovery of inter-block information gained 10 percent in efficiency. In some other experiments this plan gave little or no gain over randomized blocks, but the pair control insures utilization of a gain if it is available. The gain over complete randomization is large. It is not likely, however, that these types of randomization would be used in such experiments. The vital comparison is with the design where each test substance is compared with a standard in every trial. The incomplete-block design, here used, devoted five-sixths of the work to tests, as compared with one-half where a standard occurs in every pair. Experimental error with incomplete blocks is but little higher than that for comparison of test and standard in similar experiments using a standard in every pair. It is definitely lower than that for comparison of two test substances through the standard.

This experimental plan would seem to show real promise in certain situations of the type discussed.

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IN MEMORIAM

Forrest Rhinehart Immer
1899 — 1946

Dr. Forrest R. Immer, associate director of the Minnesota Agricultural Experiment Station, died at St. Paul, Minn., on February 2, 1946, in his forty-seventh year. In his passing the University of Minnesota lost a capable administrator and distinguished plant breeder, and biological statistics lost one of its most constructive leaders. Dr. Immer is survived by his widow, Myrtle Link Immer, and one daughter, Ruth Ann.

Dr. Immer was born in Spencer, Iowa, on July 18, 1899. While a small boy he moved with his parents, Minnie Maas Immer and Albert Immer, to a farm near Jeffers, Minn. He received his high school diploma at Windom, Minn., in 1917 and served a few months in service during the first World War. Subsequently he completed his education at the University of Minnesota, receiving the B.S. degree in 1924, the M.S. degree in 1925 and

the Ph.D. degree in 1927.

His professional advancement at the University of Minnesota was rapid. From 1927 to 1929 he was an instructor in Plant Genetics and in 1920-30 was assistant plant geneticist. In 1930 he became associate geneticist for the Division of Sugar Plants, United States Department of Agriculture. This position he held until 1935, except for the academic year of 1930-31 during which he was a fellow of the National Research Council, studying statistics with Dr. R. A. Fisher at the Rothamsted Experiment Station, England, and plant breeding at the Svalof Plant Breeding Station, Sweden. He was made associate professor at the University of Minnesota in 1935, and full professor in 1937. In 1941 he was appointed vice director of the Minnesota Agricultural Experiment Station and a year later was elevated to the post of associate-director.

During 1944, Dr. Immer took time out from his administrative duties for special service in England with the Eighth Air Force. Assigned to the Operations Analysis Section, whose duty it was to analyze bombing operations and improve bombing accuracy, he served during the air war in Europe and received citations from General H. H. Arnold and Lieutenant General J. H. Doolittle for exemplary service.

Upon his return in November 1944 to his position as associate director of the Minnesota Agricultural Experiment Station he plunged immediately into the task of fitting agricultural research to the needs of the postwar period. He was given the important posts of chairman, North Central Regional Directors, Farm Structures Committee; chairman, Association of Land Grant Colleges Committee on Farm Structures Legislative Bill; chairman, North Central Regional Directors, Poultry Breeding Committee; and member of the Crops Section of the American Society of Agronomy.

For many years Dr. Immer has been consulting editor in statistics of the American Society of Agronomy. Since 1935, he held the position of adviser in applied statistics in the Minnesota Agricultural Experiment Station.

At the time of his death, Dr. Immer was on the Editorial Committee for the Biometrics Bulletin of the Biometrics Section of the American Statistical Association. He had been very active in the formation of the Biometrics Section within the American Statistical Association in 1938.

Dr. Immer's research has been largely in plant breeding with special emphasis on statistical analysis of research data. Many graduate students at the University of Minnesota came under his direction and guidance in his course in Advanced Agricultural Statistics. Besides 51 publications, most of them in scientific journals, he is author with Dr. H. K. Hayes of a standard textbook "Methods of Plant Breeding," published in 1942.

Among the societies that honored Dr. Immer with membership are Alpha Zeta, Sigma Xi, Gamma Sigma Delta, American Association for the Advancement of Science (fellow), Genetics Society of America, American Society of Agronomy, American Statistical Association, and the Science Club of the University of Minnesota.

His integrity and unswerving devotion to the highest ideals of science, his kindness and untiring devotion to the service of others were a lasting inspiration to all those who had association with him.

E. L. LE CLERG
U. S. Department of Agriculture

WORK IN STATISTICS AT THE GEORGE WASHINGTON UNIVERSITY

All statistics at The George Washington University is directed and taught in and by the Department of Statistics. Each teacher is a specialist in statistical theory and method. Each teacher is well trained in pure mathematics and in mathematical statistics and in

addition has training in at least one field in which statistics is applied.

The theme of the members of the department is that statistics is a science and is the fundamental and most important part of inductive logic. It is our purpose to continuously strive to connect theory with operation. In our opinion, this is one of the most crying needs of the day.

The functions of the department are fourfold: (a) To teach beginning courses in statistics (non-mathematical) that are useful and desired by other departments wishing the subject merely as a tool subject; (2) To teach statistics; (3) To assist other departments with their problems and in their research where statistical method is involved; (4) To conduct and do research in mathematical and applied statistics as well as act in a consultative capacity.

The Department offers a complete program of courses as may be seen from the University Bulletin, so that a student may receive his Bachelor's degree with a major in statistics. The requirements are one year of philosophy including logic, the integral calculus and 24 credits in statistics. In addition, a student may receive the degree Master of Arts or Master of Science. Here the student is required as a prerequisite to have studied advanced calculus, differential equations and statistical mathematics. The latter includes the pertinent knowledge of theory of functions, modern algebra, n-dimensional geometry. In addition he is required to study advanced mathematical statistics, modern theories and asymptotic laws of probability, multivariate anaylsis, statistical inference, design of experiment and testing of hypotheses. A reading knowledge in one modern foreign language is required. Finally, a student may receive the Ph.D. degree in mathematical or applied statistics. Here a reading knowledge in French and German is required. For this degree, the student with guidance of a Master prepares himself in at least five fields of knowledge. They are analysis and modern algebra, mathematical statistics and probability, and one or two fields that are fields of application. This degree is given by The Graduate Council. Each year teaching fellowships are available for prospective students who are qualified to begin study for the Doctorate. The Department offers its courses during the day as well as in the evening so that a full-time student as well as an individual who is employed and wishes to study, can find an adequate program for his needs. Many of our students are full-time and others are part-time and hold jobs which bring them in contact with practical problems involving statistical method.

Our classes have many students not candidates for a degree with major in statistics. They study the subject for the sake of its applications to their major subject and vocation. Also, many students study statistics as an elective for cultural purposes. As well as serving our own students who are majoring in statistics, we serve groups of students in various categories of study and employment in statistics who are majoring in education and psychology, in the social sciences, in the languages and humanities, in philosophy, in engineering, in mathematics, in bacteriology and physiology, in the physical sciences and the biological sciences.

For the Doctorate, a consultative committee of at least five guides the work and study of the student leading to the Council Fellowship Examinations. This committee is composed of individuals who represent the respective required fields of study. The Chairman of the committee is the Chairman of the Department of Statistics. The Council Fellowship Examination consists of six written examinations each taking from four to six hours to complete properly and is given on six successive days. At the successful completion of same, the student is ready for and begins work on his dissertation under the direction of his Master, a member of the Department of Statistics, and is eligible for and becomes a Fellow of the Graduate Council.

The present members of the Department of Statistics are: C. R. Cassity, Otto Dekom, Solomon Kullback, Dorothy Morrow, A. C. Rosander, J. H. Smith, and Frank M. Weida, Chairman.

(15)

PRICE, W. C. (University of Pittsburgh). **Statistical Abstract of Measurement of Virus Activity.**

Two theories regarding the nature of infection by viruses were discussed. The most tenable hypothesis is that infection is dependent upon the chance of a single virus particle coming into contact with a susceptible region of the host. On this basis, the plot of the degree of infectivity as a function of concentration leads to a slope unity over a considerable range. The failure of virus infectivity data to conform precisely with this hypothesis can be accounted for by assuming a reversible aggregation of virus particles. In testing the relative activity of a virus sample, the experimental arrangement, therefore, is such that the slope of the infectivity curves can be calculated directly from the data. With such an arrangement, a slight modification of the procedure of Bliss and Marks can be used for estimating the activity of a virus sample and for calculating the standard error of the estimate. The accuracy of the method was tested in actual practice.

(16)

HOMEYER, Paul G. (Iowa State College). **An Analysis of Collaborative Chick Assays for Vitamin D.**

An analysis is presented for the data from a collaborative study by 31 laboratories of chick assays for vitamin D. The relative efficiencies of alternative methods of reducing the data to an estimated potency of an unknown are given. Estimates are made of the amount and sources of variation in response encountered in these assays and suggestions are given for reducing the variation.

(17)

BLISS, C. I. (Yale University). **An Experimental Design for Slope-Ratio Assays.**

When the response to a drug is a linear function of arithmetic dosage units, the relative potency of two preparations can be computed as a slope-ratio assay. Their dosage-response curves are computed by solving three simultaneous equations to obtain the common intercept a', the slope of the standard, b_1, and the slope of the unknown, b_2. The method is applicable to certain microbiological assays for the vitamins. Their calculation is simplified when such assays meet the following requirements: (1) restriction of treatments to the zone within which the response is related linearly to the dose, (2) equal spacing of doses on an arithmetic scale beginning with the negative control, (3) an equal number (k) of doses of standard and of each unknown and (4) one tube for each dose of unknown and h tubes for the negative control and for each dose of the standard.

With this design it can be shown that

$$a' = \frac{2(2k+1)S(y) - 6T_{xy}}{N(k-1) + 3h(k+1)}$$

where N is the total number of responses in the assay, $S(y)$ is the sum of all responses and T_{xy} is the sum of the products over all m preparations of each response multiplied by its coded dose, 1 to k. The slope of the standard is computed as

$$b_1 = \frac{3}{2k+1}\left\{\frac{2S(xy_1)}{hk(k+1)} - a'\right\}$$

where $S(xy_1)$ is determined from the standard. The slopes for the unknown are obtained similarly with $h = 1$. In terms of coded doses, relative potency is computed as

$$J_2' = \frac{b_2}{b_1}$$

which is decoded by multiplying by i_s/i_u, the respective intervals between successive doses for the standard and for the unknown. By large sample theory the sampling variance of J' is

$$V(J') = \frac{6s^2}{(2k+1)b_1^2}\left\{\frac{h+J'^2}{hk(k+1)} + \frac{3(1-J')^2}{N(k-1)+3h(k+1)}\right\}$$

where

$$s^2 = \frac{S(y^2 - a'S(y) - b_1S(xy_1) - \ldots - b_pS(xy_p)}{N-p-1}$$

(18)

PATTERSON, R. E. (Texas Agricultural Experiment Station). **The Use of Adjusting Factors in the Analysis of Data with Disproportionate Sub-class Numbers.**

A method of adjusting is described whereby the sums of squares for the various sources of variance can be eliminated. When this method of adjusting is applied to data with unequal subclass numbers, it is possible to obtain a sum of squares for each source of variance that is free from the influence of the other effects. The process of adjusting is accomplished by substituting in the following equation:

$$X_{rs} - X_s + X = \bar{A}_{rs}$$

where X_{rs} is the mean of the *r*th subclass in the *s*th row or column, X_s is the mean of the *s*th row or column, X is the grand mean and $\bar{A}_{rs}$ is the adjusted mean of the *r*th subclass in the *s*th row or column.

The method is based upon the assumption that the weighted sum of squares of the subclass means that are adjusted for the border mean effects is an efficient estimate of the variance due to interaction. Justification of this assumption is indicated by the fact that the difference between the differences of subclass means for a given classification is unchanged by the adjusting process. It is further demonstrated that if a sufficient number of adjustings are carried out the results will be the same as those obtained by the method of fitting constants.

(19)

MOOD, A. M. (Iowa State College). **Selection of Sample Sizes for Detecting Treatment Differences.**

In designing experiments to study treatment differences, there is often available an estimate of the error variance obtained from a preliminary experiment or from a previous experiment with similar materials. Using this estimate and its number of degrees of freedom, it is possible to compute the number of degrees of freedom necessary in the error variance of the projected experiment to detect specified difference means. A table has been computed whereby one may readily determine the number of degrees of freedom so that the probability will be .80 that a specified treatment difference will be detected at the .05 level of significance.

(20)

DELURY, D. B. (Virginia Polytechnic Institute). **The Analysis of Latin Squares When Some Observations Are Missing.**

The discussion in this paper is given explicitly for a biological array which employs a 4 x 4 latin square in several replications. However, the methods are easily adapted to any latin square and to various other designs as well.

Methods of analysis, when some observations are missing, are discussed for the following cases.

(1) One or more "single" observations are missing.

(2) Several columns are missing.

(3) One column is missing.

(4) Two columns are missing.

(5) One column and one or more single observations are missing.

In all of these cases, with the possible exception of (5), methods of analysis have been available for several years. However, it is believed that the approach used in this paper leads to considerable simplification in most cases.

Cases (1), (3) and (5) may be treated by means of formulae alone; cases (2) and (4) require the solution of normal equations. Simple systematic methods of setting up these normal equations are developed.

(21)

HARSHBARGER, Boyd. (Virginia Agricultural Experiment Station). **Rectangular Lattices.**

The paper presents an extension of the incomplete block design to the case when the number of varieties or treatments is expressible as the product of any two integers with an explicit solution for the case where the number is k(k — 1). The varieties are adjutsed by both the inter- and intra-block information. The name Rectangular Lattice is proposed for the design since the word lattice carries no implications of squareness. The analysis for the Rectangular Lattice is derived by the method of fitting constants. This is a departure from the method used in the square

lattice which was based on the analogy with a confounded factorial.

The construction of the Rectangular Lattice and the arrangement of the varieties in the field, the numerical computations for the analysis of variance, the adjustment of varieties using the recovery of inter-block information, the calculations of the standard errors for testing the significance of the differences between variety averages, and the efficiency of the design relative to a complete randomized blocks are all given in detail.

The application of the analysis is illustrated by applying it to variety tests; however, the block has wide applications to other types of problems. Fertility or biological treatments may be substituted in place of varieties and the procedure is the same. With slight modifications the analysis can be used to measure the efficiency of laboratory personnel and to estimate the individual biases in measurements. In engineering the possibilities are great.

QUERIES

QUERY: We had a problem last spring in testing a large number of top crosses. Where you are going to have a group of crosses running into the hundreds, what are you going to do? *(Query 1)* Any design which you may use is going to involve a large area and the accuracy of the results might be questionable. We have a planting of a 12 x 12 triple lattice which is also arranged so that a check plot occurs every third plot. It seems to us that this system of check plots give more reliable results, especially where such factors as lodging is as important as yield. Is this correct? *(Query 2)*

ANSWER: *(Query 1)* "Where you are going to have a group of crosses running into the hundreds, what are you going to do?" For such experiments, various designs have been proposed, the most extensively used being the incomplete block designs devised by Yates. In the balanced designs, all variety comparisons are made with equal accuracy, as for example in the balanced lattices and Youden squares. Rather severe restrictions on numbers of replications in these balanced designs have led to the wide use of partially balanced designs such as the simple, triple and cubic lattices. References: Yates—Annals of Eugenics 9:136 (1939) and 10:317 (1940); Journal of Agricultural Science 30:672 (1940); Cox, Eckhardt and Cochran—Iowa Agricultural Experiment Station Bulletin 281 (1940).

(Query 2) "Is this correct?" In answer, I quote from a mimeographed text on Experimental Designs by Cochran and Cox: "The method of systematic controls is very flexible since it can be used with any number of treatments and any number of replicates. Little evidence is available about the increase in accuracy obtained from the controls, though it seems probable that the increase is seldom large if the extra space occupied by the controls is taken into account. The calculation of the best adjustments . . . is rather tedious, while if a crude type of adjustment is made most of the potential advantages of the method may be lost."

In the Journal of Agricultural Science 26:424 (1936), Yates expressed the opinion that the newer designs (mentioned above) are likely to be more efficient than randomized blocks with systematic check plots. If, for example, querist had used a 10 x 10 simple lattice design for his 96 top crosses with 4 checks or standards, he could have gained an extra replication and at the same time could have saved 32 plots.

In either design, comparisons among lodging percentages are of the same reliability as those among yields.

In his 1936 article, Yates gave the appropriate statistical analysis for systematic check plots in randomized blocks. So far as I know the analysis for systematic checks in a triple lattice has not been published. Of course, the experiment may be looked upon as randomized blocks to which the method of Yates is applicable; but the anticipated advantage of the triple lattice may have then been lost.

—*Walter T. Federer*

QUERY: One column in the table of analysis of variance is designated by some as Mean Square and by others as Variance. Which is right?

ANSWER: There is no categorical answer to your question. Apparently, different people emphasize different features of the column. It may suffice to cite three of these features.

1. If 5 subsamples, each with 10 items, are drawn at random from a normal population with variance, σ^2, the analysis of variance is:

Source of Variation	*Degrees of Freedom*	*Mean Square*
Means	4	M_1
Items	45	M_2
Total	49	M_3

Here, each M is an estimate of σ^2, and the column might well be headed, Estimates of Variance.

2. If 4 male pigs are taken at random from each of 10 litters from a random selection of dams of a common breed and having the same sire, the analysis of variance is:

Source of Variation	*Degrees of Freedom*	*Mean Square*
Dams	9	M_1
Pigs	30	M_2

It may now be assumed that M_2 is an estimate of σ^2 in a normal population of male pigs undifferentiated by either sire or dam. But M_1 is not an estimate of that same variance—a component due to dams has been added. If this added component is denoted by σ_D^2, then M_1 is an estimate of

$$\sigma^2 + 4\sigma_D^2$$

This quantity is not the variance of either pigs or dams, nor is it the variance of litter means, the latter being $\sigma^2/4 + \sigma_D^2$. It may seem clearer, therefore, to distinguish its estimate by some such term as Mean Square. But if this term is put in the column heading it covers M_2 as well, and so some object to it. Perhaps the heading should be, Mean Square and Variance.

3. In ordinary experimentation treatments are not chosen at random; hence, it is not informative to estimate the component of variance, σ_M^2. Still less is it useful to look upon M_1 as representing any variance pertinent to the experiment. The goal is reached by computing $F = M_1/M_2$, and the column might be headed, Terms of F. On the other hand, Fisher calls M_1/M_2 the variance ratio, indicating the fact that both terms of the fractions are variances.

In the foregoing some of the complications are ignored, but enough has been said to show why I cannot give a definite answer to the question. Until some consensus is reached by the statisticians, laymen may well follow their preferences. George W. Snedecor

QUERY: I wish to learn if the growth in height of the cotton plant may be expressed by Robertson's growth equation,

$$\log \frac{x}{x - a} = k\,(t - t_1),$$

where x is the height of the plant at any time, t; a is the maximum height attained; and t_1 is the time at which $x = a/2$.

I measured the height each 5 days during the growing season. A value of k was calculated by substituting successive pairs of x and t in the equation, then averaging the k's. From this average k, I calculated the theoretical values of x. In testing the fitness between the theoretical and observed values of x by means of chi-squares, how should the degrees of freedom be computed?

ANSWER: It is not usual to test the goodness of fit of a measured variate by chi-square. The formula

$$\chi^2 = \frac{\Sigma\ (\text{Observed} - \text{theoretical})^2}{\text{Theoretical}}$$

applies only to *frequencies* and not to values of such a variate as height.

The fitness of your measurements to Robertson's formula may be tested graphically by plotting values of $\log x/(x - a)$ against the corresponding $(t - t_1)$. The points should indicate a straight line passing through the origin and having the slope, k.

If the graph shows a reasonably good fit, you may wish to make more precise tests by use of regression. Linear regression fitted to your plotted points is satisfactory for most practical purposes: for very exacting requirements, the method of least squares may be applied directly to the original formula.

George W. Snedecor

QUERY: A formula often seen for the mean square "between groups" is $(S_1 - S_2)^2/2k$ where S_1 and S_2 are the sums in the respective groups and k is the number of individuals in each. This is true for two groups of unpaired variates. If the variates are *paired*, is the correct formula for mean square $(S_1 - S_2)^2/k$?

ANSWER: The answer depends on the unit used in computation.

If querist is using differences between paired observations as his unit or computation, as is done in applying the ordinary t-test to unique samples, and if he then prefers to make the F-test, his second formula is the correct one. The reason is that, under the conditions usually assumed, his differences estimate twice the variance of the original observations; hence, the corresponding formula for mean square between group means is $(S_1 - S_2)^2/k$.

But if the unit of computation is the individual observation, as is customary in analysis of variance, then the first formula for mean square is the correct one for both group and individual comparisons.

David B. Duncan

QUERY: In an article by Frank Wilcoxon in the December issue of the Biometrics Bulletin the statement is made that "Table II shows that the total 3 indicates a probability between 0.024 and 0.055 that these treatments do not differ." Although I am not a mathematician, it seems doubtful to me that the author has evaluated the probability stated, especially since the table heading indicates a different probability. Will you set me straight on this?

ANSWER: The statement in question should read, "The probability of obtaining a total of three or less, under the assumption that the treatments do not differ, lies between 0.024 and 0.055, as is indicated by Table II."

If the treatments do not differ, it would be expected that the rank total of one sign would be distributed about 18 in repetitions of such an experiment. There is a definite probability of chance occurrence of any possible total of one sign or a lesser total, under the assumption that the treatments do not differ. If this probability is sufficiently small, the assumption that the treatments do not differ is abandoned, and it is concluded that the treatments differ. In calculating the probabilities it is necessary to double the probability for a total of one sign, since an unlikely result may arise either through a small negative total or through a small positive one.

Frank Wilcoxon

NEWS AND NOTES

The Biological Methods Group of the Society of Public Analysts and Other Analytical Chemists held its first annual general meeting on Monday, February 25, and the provisional elections of A. L. BACHARACH as Chairman, and ERIC C. WOOD as Honorary Secretary, at the inaugural meeting of last October were confirmed. Mr. Eric C. Wood, Virol Limited, London reports, "The formal business was followed by the reading of papers by N. T. GRIDGEMAN on 'The transformation of metameters with special reference to vitamin D assays,' and by E. C. FEILLER, entitled 'Some remarks on the statistical background of bio-assays.' Mr. Gridgeman is a bio-chemist with Lever Bros. and Unilever, Ltd., the large Oils and Fats Combine, while Feiller has recently been appointed as statistician to the National Physical Laboratory. Both speakers dealt very interestingly with the question of transforming measurements of variates into other functions for the purpose of normalizing distribution, equalizing the variates, or simplifying the computations. Quite a lively discussion took place afterwards." We would like to hear from groups in England or other foreign countries regarding their statistical activities . . . It is no longer necessary to feel sorry for D. J. FINNEY as conditions have improved and he is now more comfortably situated. In fact, he says that he is getting down to serious work once more, and is giving an elementary course on "Statistical methods in scientific research" . . . The Statistical Department at Rothamsted Experimental Station now includes FRANK YATES, D. A. BOYD, O. KEMPTHORNE and J. W. WEIL. We are counting on that visit to the States

before long, Mr. Yates . . . JOHN WISHART, Department of Statistics, School of Agriculture, University of Cambridge, is interested in linking mathematical and biological schools in the study of statistics . . . A. C. FABERGE, formerly a biologist and geneticist at the Galton Laboratory, London now occupies the position of Research Associate in the Botany Department at the University of Wisconsin, Madison. He gave us considerable information regarding the statisticians in England. Some of the news will be reported later. Mr. Faberge says, "W. L. STEVENS, after spending three years in Portugal teaching mathematical statistics at Coimbra on behalf of the British Council, has taken a job as statistical advisor to Imperial Chemical Industries, Billingham, Yorks . . . The Galton Professorship has been given to L. S. PENROSE . . . The physiologist H. KALMAS, who worked with J. B. S. HALDANE'S department, has joined Professor Penrose in the position that I held . . . M. S. BARTLETT has returned from war work to Cambridge recently" . . . A. BRADFORD HILL has succeeded MAJOR GREENWOOD in the chair of medical statistics in the London School of Hygiene and Tropical Medicine at the University of London. Professor Greenwood has been made Professor Emeritus . . . DR. E. A. CORNISH, Commonwealth of Australia, Council for Scientific and Industrial Research, Section of Mathematical Statistics, University of Adelaide, South Australia, writes "During the last few years our work has increased considerably and long ago reached the stage where we could not cope with the influx. Increases of staff were demanded but I had great difficulty in securing the services of even two partly trained men, because the Universities here were doing little toward preparing people for statistics as a profession. In a desperate bid to get staff, the Council agreed to let me undertake the job of supervising courses in the University of Adelaide, so we started at the beginning of the current academic year. It was an uphill fight writing two courses of lectures, preparing exercises for practical classes, and trying to carry on with research at the same time. Fortunately, our efforts have been rewarded to some extent because we gained one really good recruit, and at the end of next year, after further intensive effort, we hope to appoint six more" . . . ERNEST E. BLANCHE has recently returned from Italy where he taught at the U. S. Army University at Florence. His new War Dept. assignment is with the Control Division, Army Service Forces, Pentagon . . . L. N. HAZEL formerly with Western Sheep Breeding Laboratory, U.S.D.A. Dubois, Idaho, is now with Kimber Poultry Breeding Farm, Niles, California . . . MAJOR WARREN H. LEONARD writes from Tokyo, "I could really write you an article on how the Japanese fail to use modern statistical methods." He is Chief of the Agricultural Division which has four branches. S. C. SALMON is head of the Agricultural Research Branch. His task is to investigate the experimental work in Japan and see that it has been converted back to peaceful ways. Mr. Leonard states, "Incidentally, there are from 375 to 400 experimental stations, branch stations, laboratories, and demonstration farms in Japan proper." V. R. BOSWELL is also working with this group . . . MAJOR A. L. FINKNER, who visited Mr. Leonard in December, is now back with the Bureau of Agricultural Economics in Raleigh, conducting cooperative work with the Institute of Statistics — a married man now! . . . You have not been forgotten down there in Mexico, E. J. WELLHAUSEN. Your letter presenting a "new source of error" has received consideration by the young reasearch workers here. I do believe some of them want to help you try to solve that problem. Mr. Wellhausen reports, "I think I can justly claim to be the first person to have used any of the modern experimental designs in Mexico. They will do the job here too. This year we planted in different parts of Mexico 45 simple lattice experiments in corn yield tests each containing 49 varieties, 4 replications, 24 hill plots. The last month (Dec.) I have been busy harvesting corn. In one section of Mexico, near Cortazar, we employed women as harvesters. In these experiments I discovered a new source of error which even at my old age rather embarassed me . . . (censored)." . . . Recent appointments to the staff of the Institute of Statistics at Raleigh include H.

L. LUCAS from Cornell University, R. J. MONROE released from the Army, and W. G. COCHRAN from Iowa State College. Mr. Cochran is an Associate Director of the Institute.

Material for the BULLETIN should be addressed to the Chairman of the Editorial Committee, Institute of Statistics, North Carolina State College, Raleigh, N. C., material for Queries should go to "Queries," Statistical Laboratory, Iowa State College, Ames, Iowa, or to any member of the committee.